Predator-Prey Analogs for Saturn's Non-Linear Ring Dynamics

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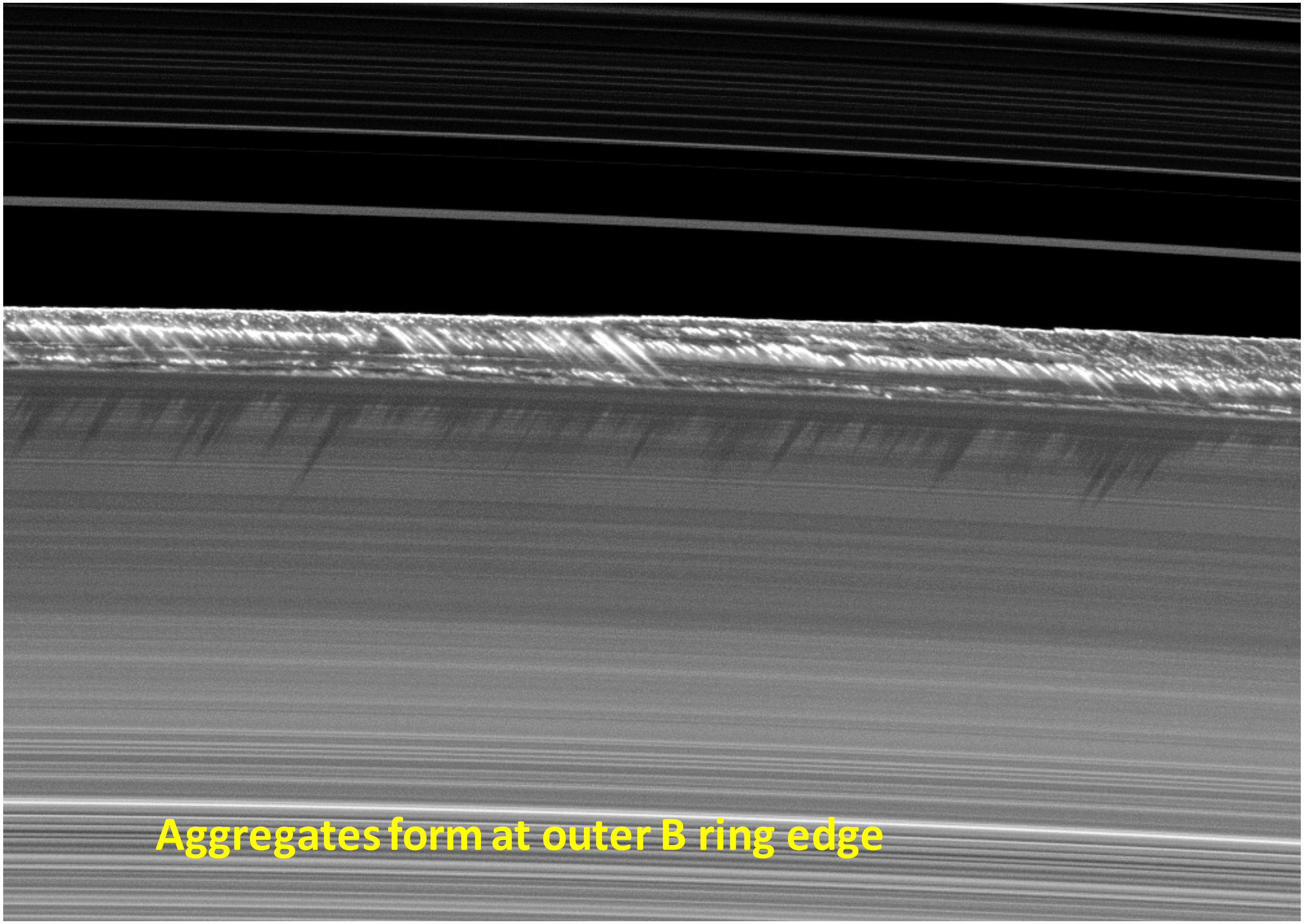
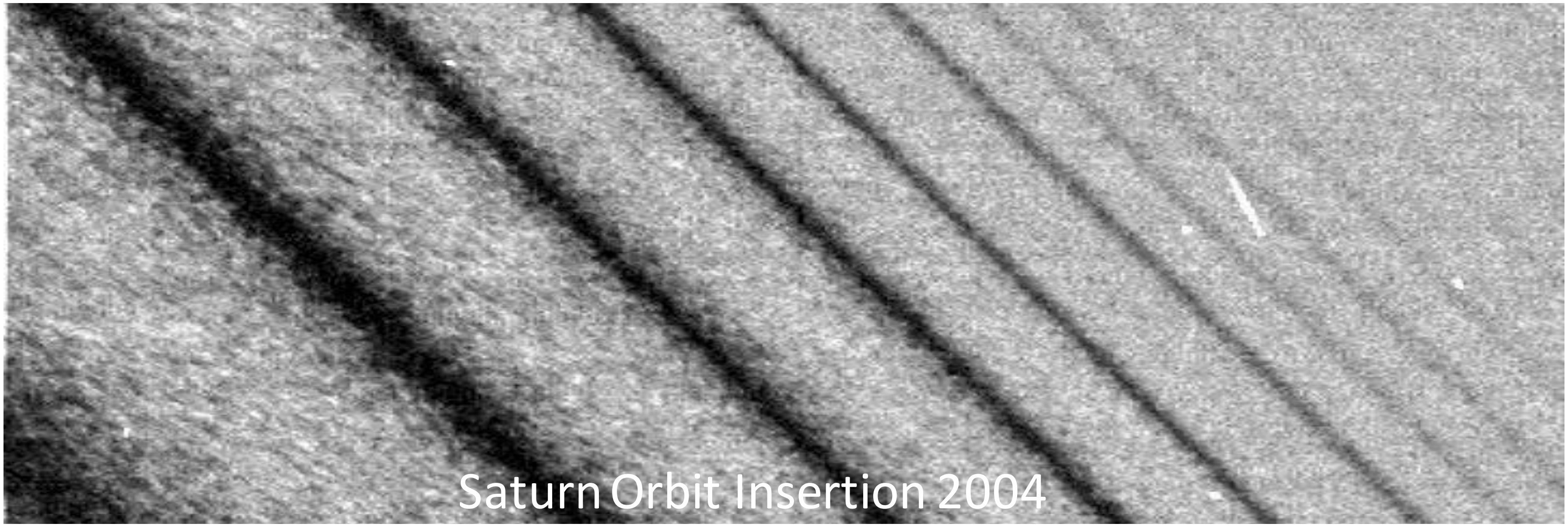
Transient ring structures appear where the rings are perturbed strongly

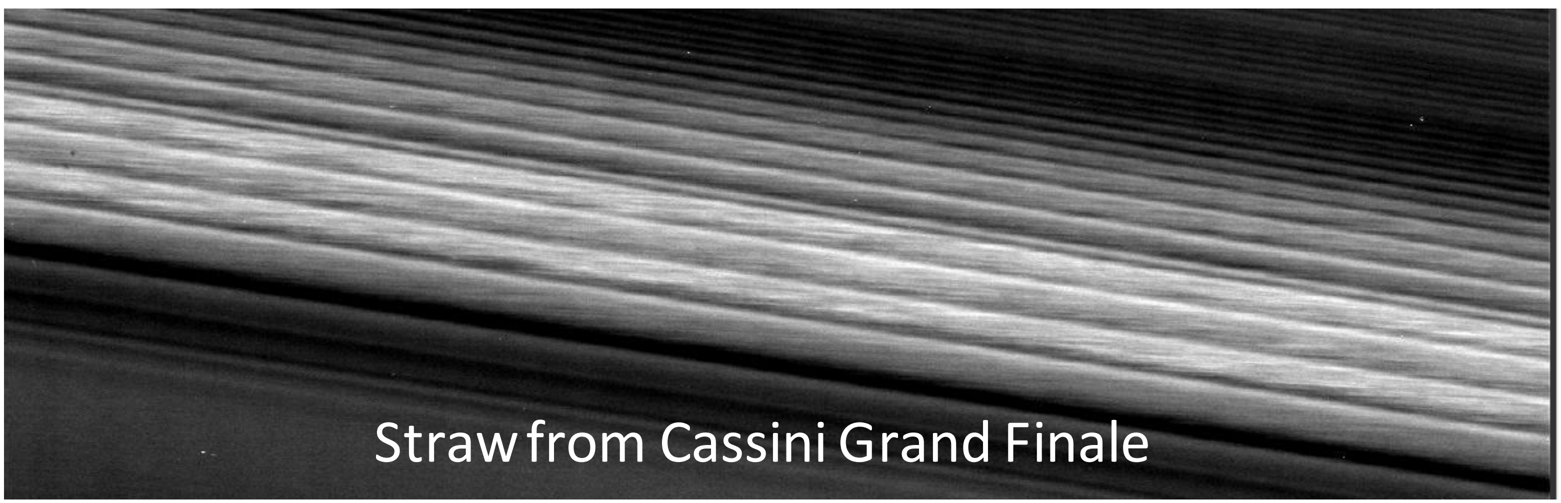
- Equinox objects at the Mimas 2:1 resonance
- Straw between density wave crests
- Excess variance increases between wave crests
- Gap edges when a moon passes by
- Solitary waves where the Janus resonance falls on Epimetheus density wave every 8 years


Aggregates form at outer B ring edge

‘Straw’ seen between density wave crests must form in less than 10 hours

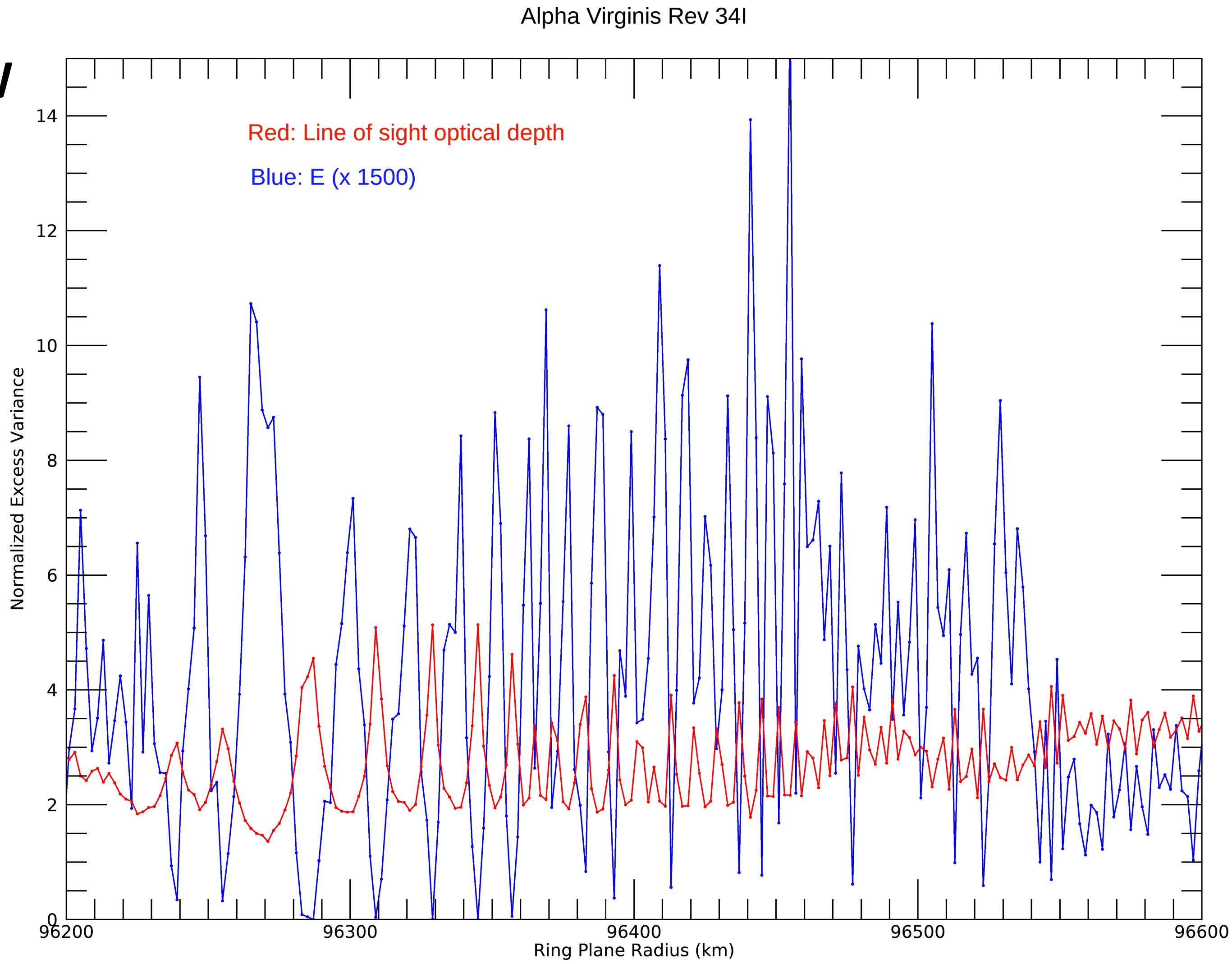


Saturn Orbit Insertion 2004



Straw from Cassini Grand Finale

Janus 2:1 DW

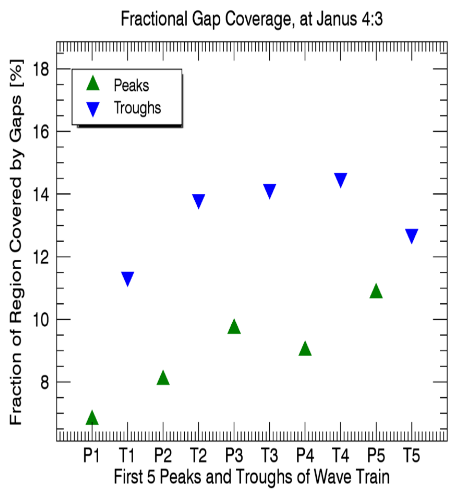


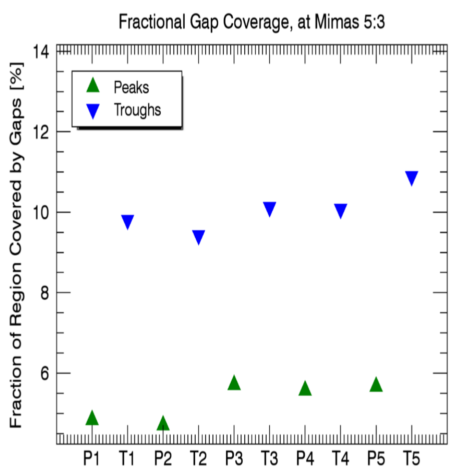
Particle statistics show larger structures between density wave crests

Gap % Coverage Trends

Trend: gaps cover more linear area in **troughs** for Janus 4:3, Mimas 5:3

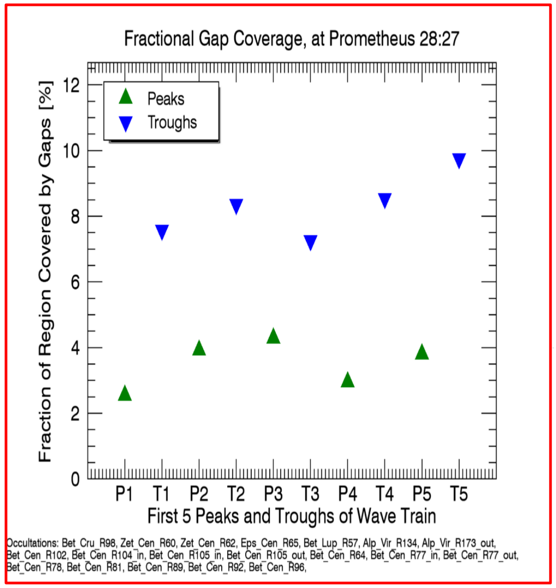
(t significances: **1e-6,8e-12**)





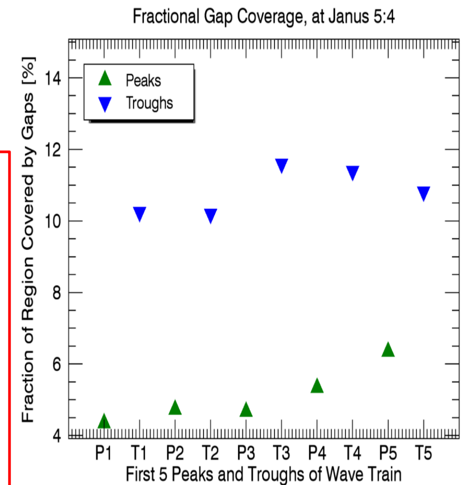
Trend: gaps coverage is greater in **troughs** for Prometheus 28:27

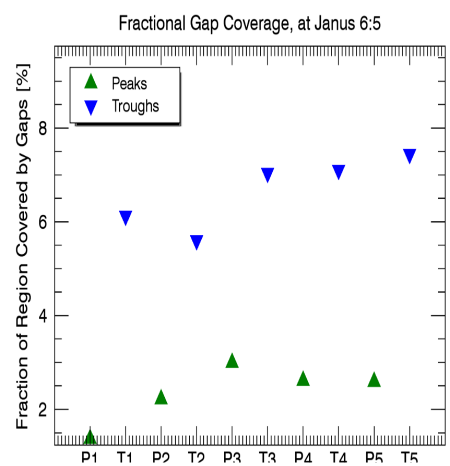
(t significance: **5e-13**)



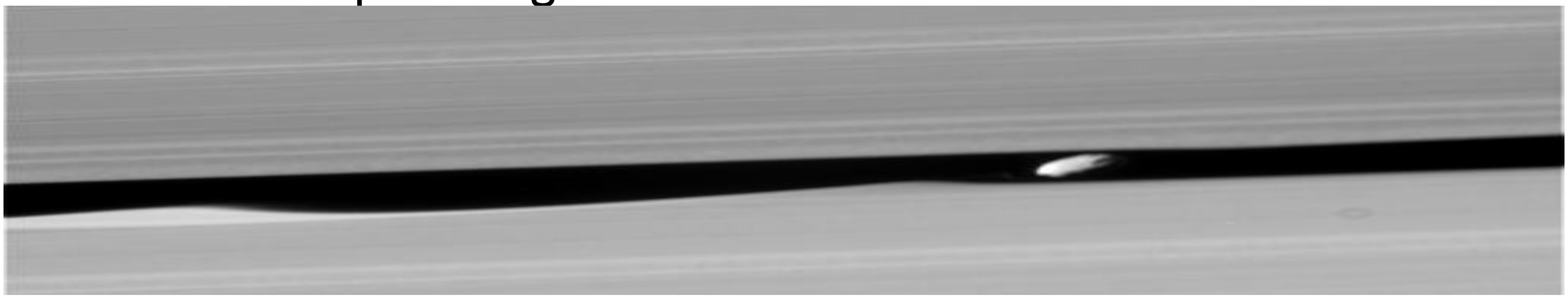
Trend: gaps cover more linear area in **troughs** for Janus 5:4 and 6:5

(t significances: **5e-16,1e-15**)



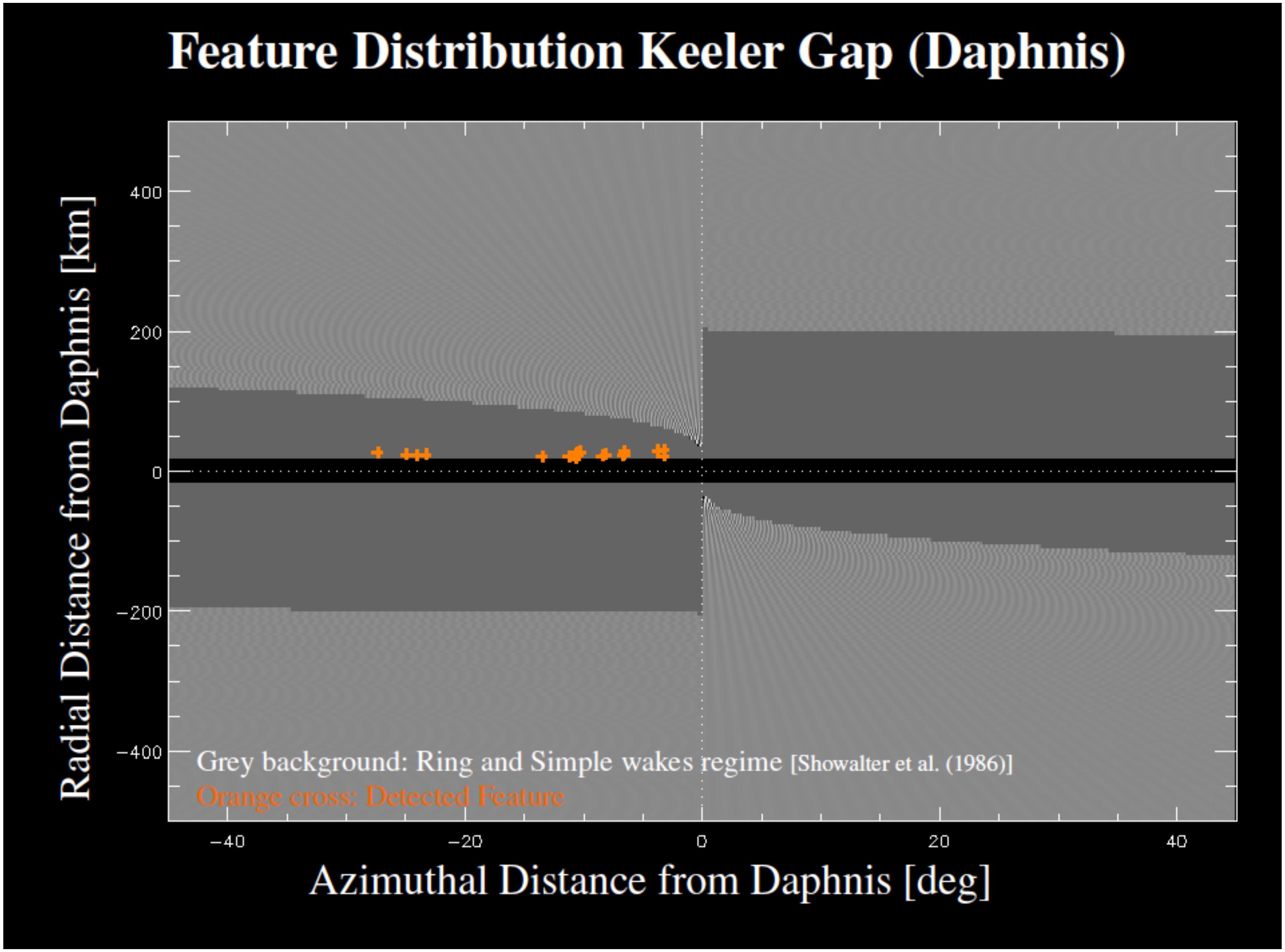


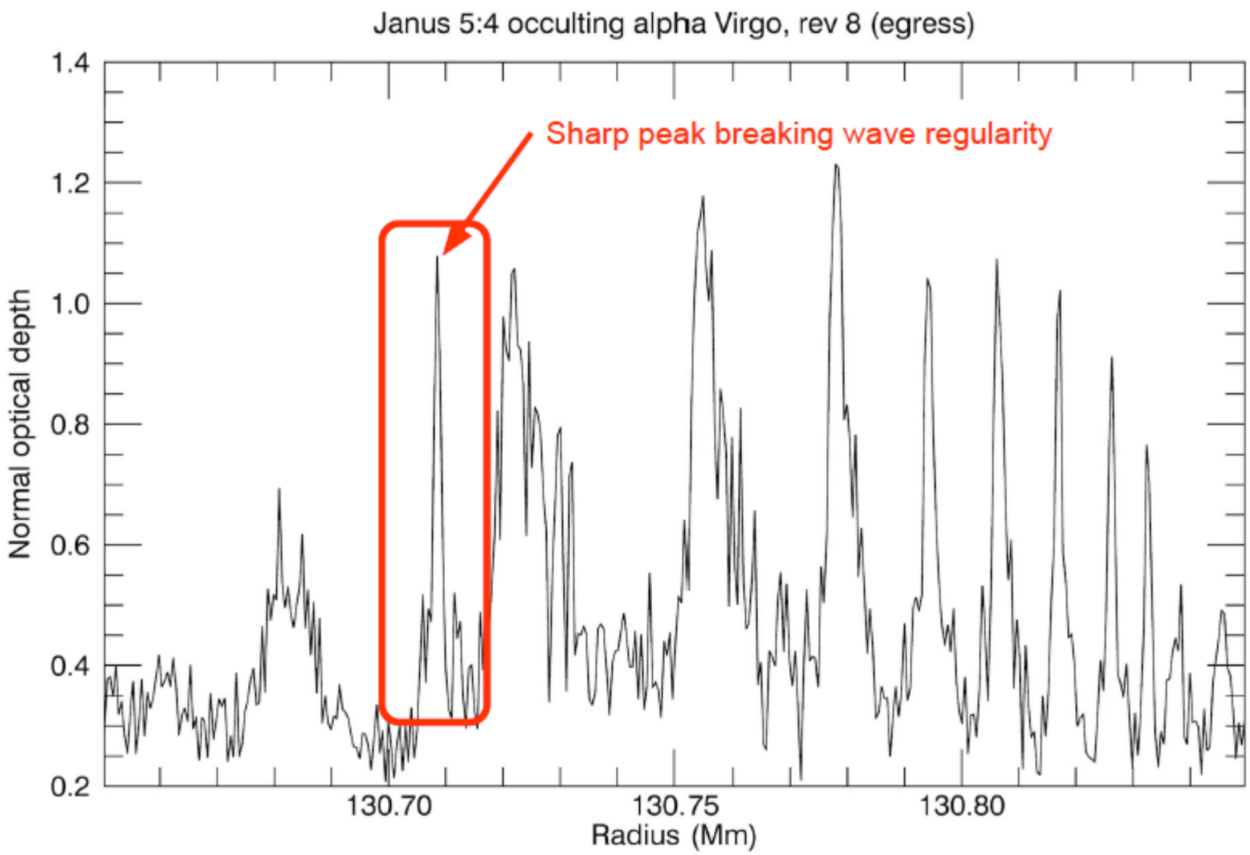
Daphnis Edge Wake shows downstream effect



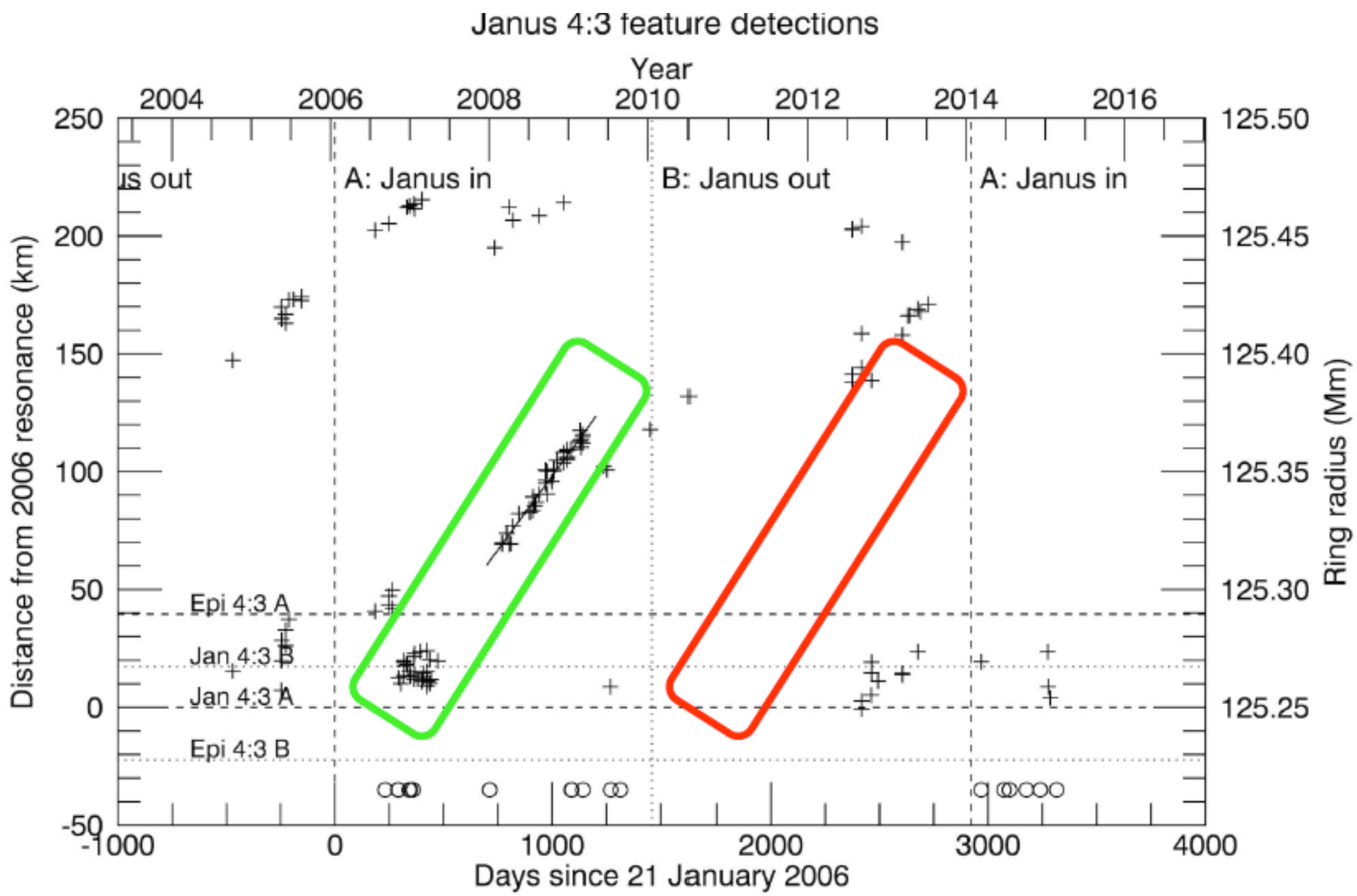
Clumps Form

Ring Edge Shears and Separates


Feature Distribution Keeler Gap (Daphnis)
Radial Distance from Daphnis [km]
400
200
0
−200
−400
−40
−20
0
20
40
Azimuthal Distance from Daphnis [deg]
Grey background: Ring and Simple wakes regime [Showalter et al. (1986)]
Orange cross: Detected Feature



Another non-linear phenomenon:
A soliton excited by Janus-Epimetheus swap, every 8 years



Solitary wave propagating through A ring, seen every 8 years

How to explain this dynamic structure?

- Solitary waves and the large amplitude, rapidly growing transient structures indicate non-linear phenomena
- N-body simulations are too slow, and don’t include all the physics
- Use a simpler model with an ecological analogy: *Predator–Prey*
- Track the mass and velocity dispersion: Relative velocity is stirred up by clumps, velocity disrupts clumps
- Force the system by the moon’s gravity driving the surface mass density and the velocity dispersion
- Allow for disk instability, using Toomre’s dispersion formula
- Use numerical simulation results for outcomes of stochastic collisions (Hyodo & Ohtsuki; Leinhardt & Stewart)

Predator-Prey Equations for Ring Clumping (Esposito *et al* 2012)

$M = \int n(m)\, m^2\, dm \,/\, \langle M \rangle;$

$V_{rel}^2 = \int n(m)\, V_{rel}^2\, dm \,/\, N$

$dM/dt = \quad M/T_{acc} \qquad - V_{rel}^2/v_{th}^2\; M/T_{coll}$

[accretion] *[fragmentation/erosion]*

$dV_{rel}^2/dt = -(1-\varepsilon^2)V_{rel}^2/T_{coll} + (M/M_0)^2\, V_{esc}^2/T_{stir}$

[dissipation] *[gravitational stirring]*

$- A_0 \cos(\omega t)$ *[forcing by streamline crowding]*

The aggregate mass, M is the 'prey'; The dispersion V_{rel}^2 is the 'predator':
It feeds off of the mass by grav stirring; The predator reduces the prey by erosion

Predator Prey Model with Logistic Growth

Basic equation:

$$T_S = \frac{T_{orb}}{2\pi\tau'_S}$$

$$\tau_s' = \tau_s\left(1 - \frac{M}{M_L}\right)$$

$$\frac{dM}{dt} = \left[\frac{M}{T_S}\frac{S}{R}\frac{\rho_0}{\rho} + \frac{M}{T_{DI}} - \frac{M}{T_S}\frac{S}{R}\frac{\rho_0}{\rho}\frac{V_{rel}^2}{V_{th}^2}\right]$$

$$\frac{M}{T_{DI}} = 0 \; when \; \omega^2 \geq 0$$

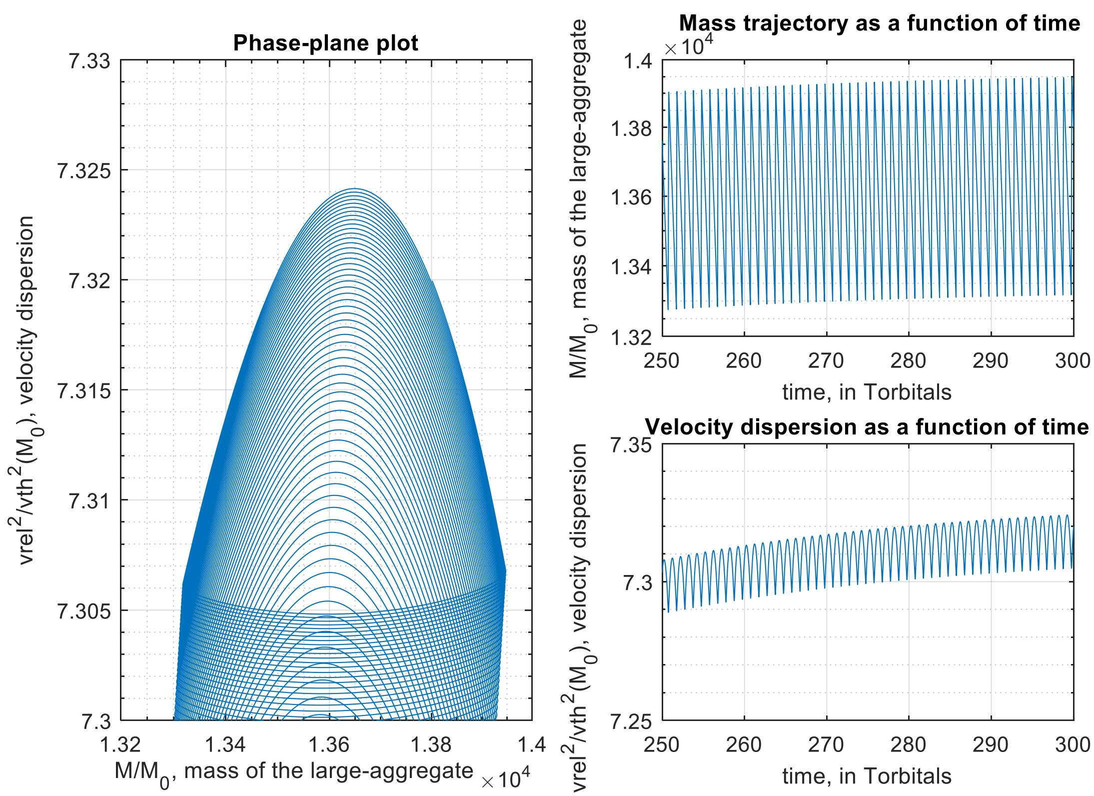
$$\frac{dV_{rel}^2}{dt} = -\frac{V_{rel}^2}{T_S}\left[(1-\varepsilon^2) + \frac{\tau_B}{\tau'_S}\right] + \frac{V_{esc}^2(M)}{T_S}\frac{\tau_B}{\tau'_S}$$

Motivation: When their Mass reaches a limiting value, the aggregates cannot grow further by ring particle sweep up, since we have only a finite number of ring particles to stick to the growing aggregates. This is modelled by adding a ***logistic growth*** term limiting the optical depth of smaller aggregates.

Thus, the closer the aggregate mass M to M_limit, the slower the growth rate.

Note: After the limiting mass is reached, the aggregates change in mass only due to stochastic collisions which yield accretions and disruptions.

Strength regime surface mass density forcing phase plots



4% variation

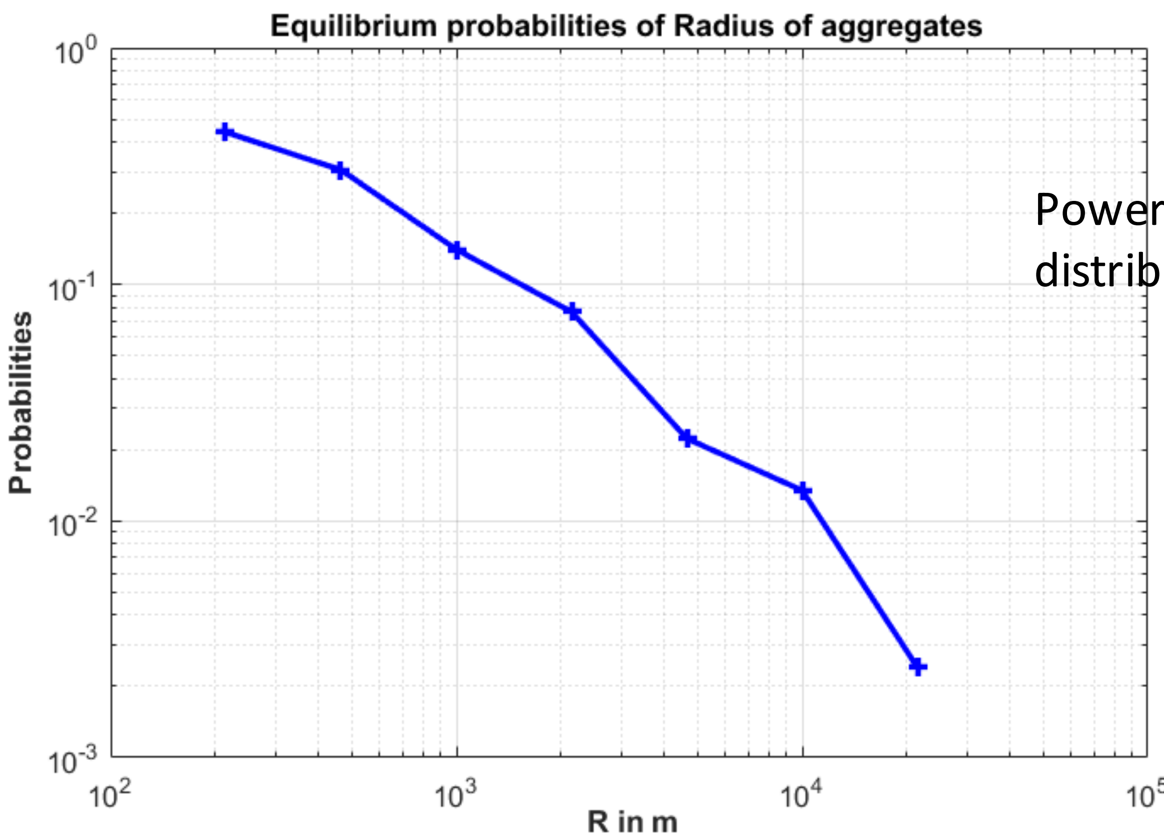
$$\Sigma = \Sigma_0 \frac{1}{\left(1 + m \sin\left(\omega_{syn} t\right)\right)}, m = 0.97$$

Disk instability when:

$$\omega^2 = \kappa^2 - 2\pi G \Sigma |k| + v_s^2 k^2$$

$$\omega^2 < 0$$

Equilibrium distribution of aggregates from stochastic collisions is a power law



Power Law index for radius distribution: -1.0158

Conclusions

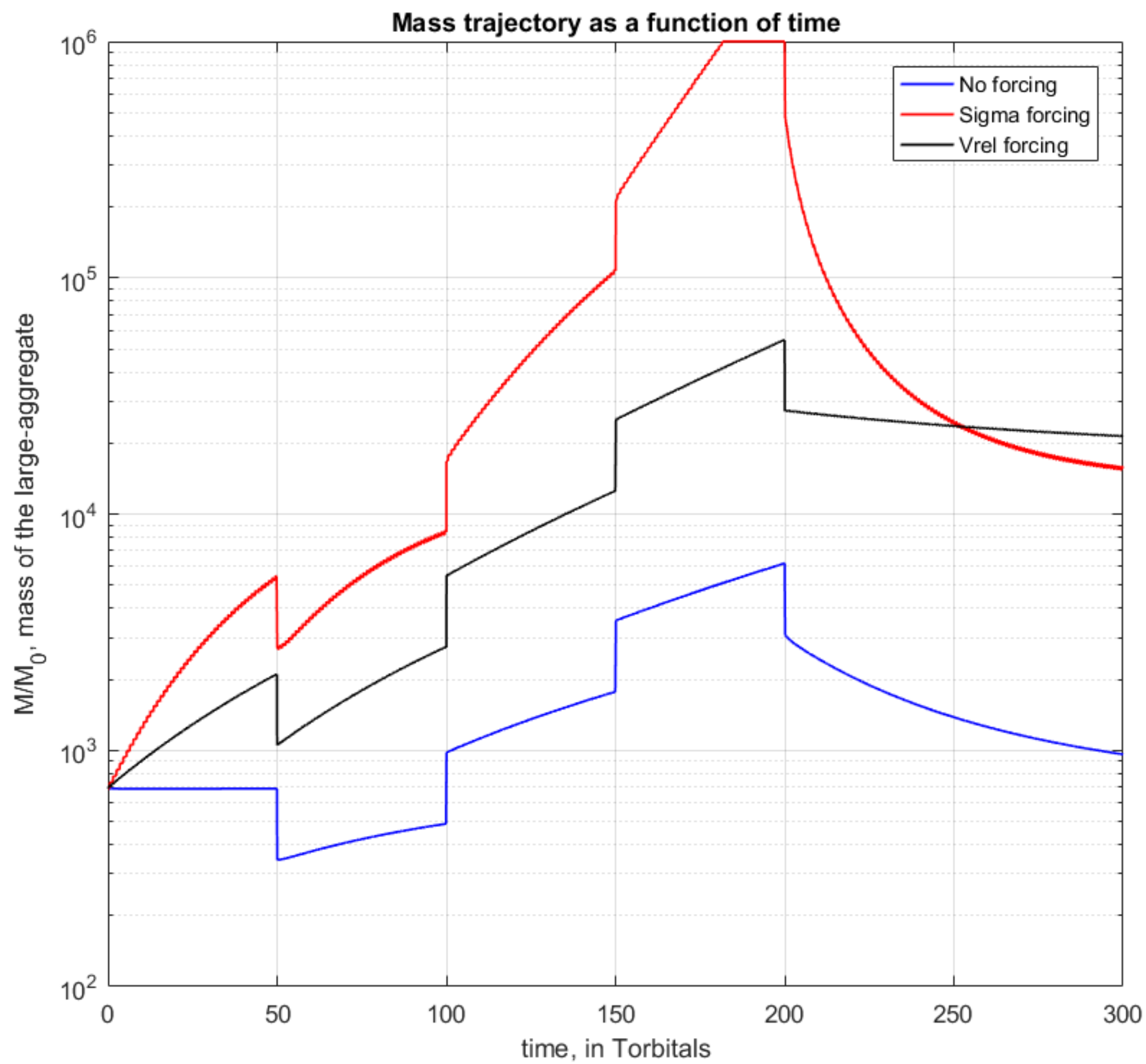
- Ring structure shows transient clumping in perturbed regions: We conclude that forcing by the moon triggers aggregation. This also increases the relative velocity, liberating small particles
- The structure forms rapidly, on orbital time scales, out of phase with the moon… and downstream of the moon's wake initiation
- We find that growth by sweep-up is too slow to explain the excess structure observed in between density wave crests
- Gravitational disk instability *can* act on orbital time scales; We use Toomre's stability parameter Q to estimate the growth rate for clumps
- We achieve rapid growth by modulating the surface mass density, decreasing the velocity dispersion or by decreasing the shear
- Aggregates from stochastic collisions have a power-law size distribution

Take away message: *Moon forcing drives accretion, triggers disk instability, producing transient clumps downstream: A continuing process of Cosmic Recycling*

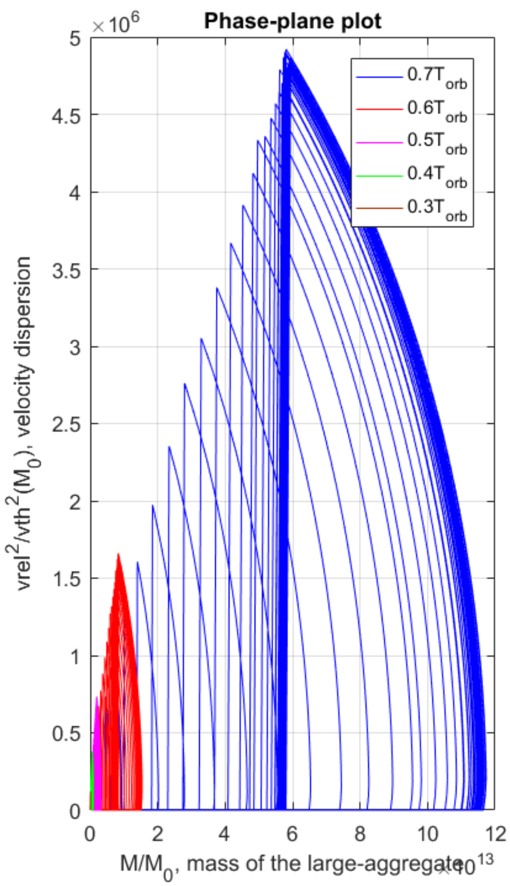
Back-Up

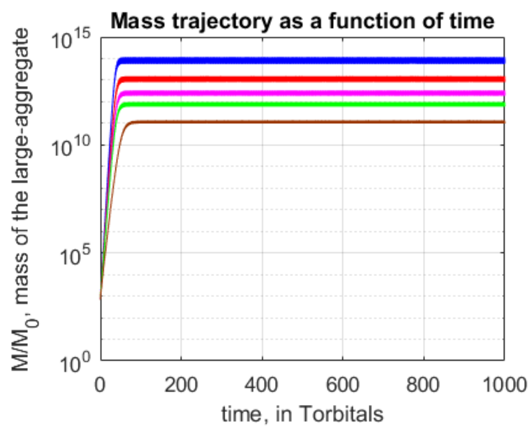
$M=0.97$
$V_{rel}=0$ for $0.01t_{orb}$

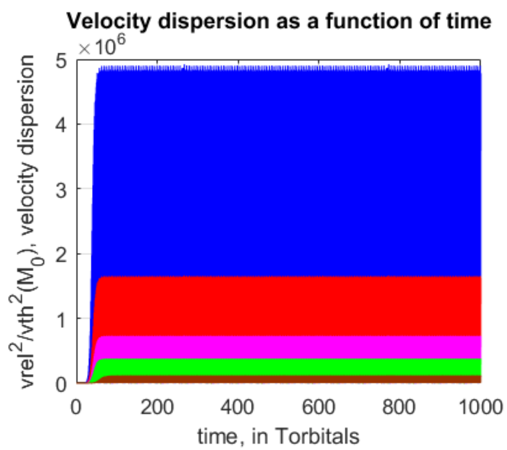
We can force the Predator-Prey model by surface mass density or by velocity variations, which give similar outcomes.



Clump mass M, from V_{rel}=0 for > $0.3T_{orb}$







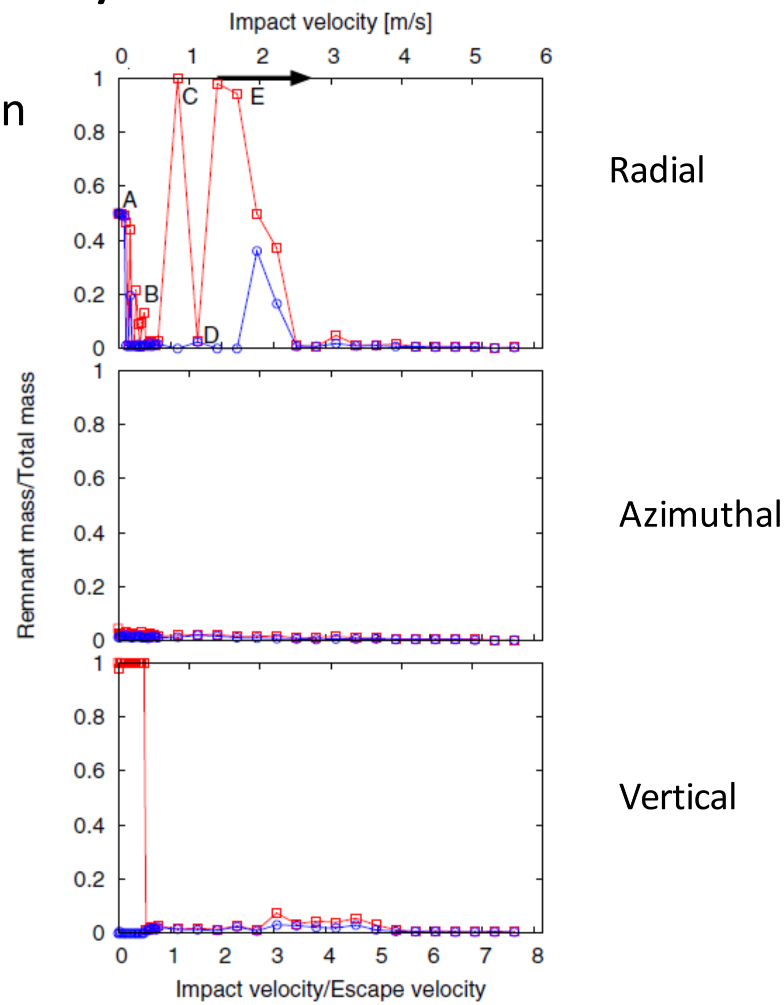
Mass variations around the fixed points:
0.7torb = 64.4%
0.6torb = 60%
0.5torb = 51%
0.4torb = 48%
0.3torb = 31%

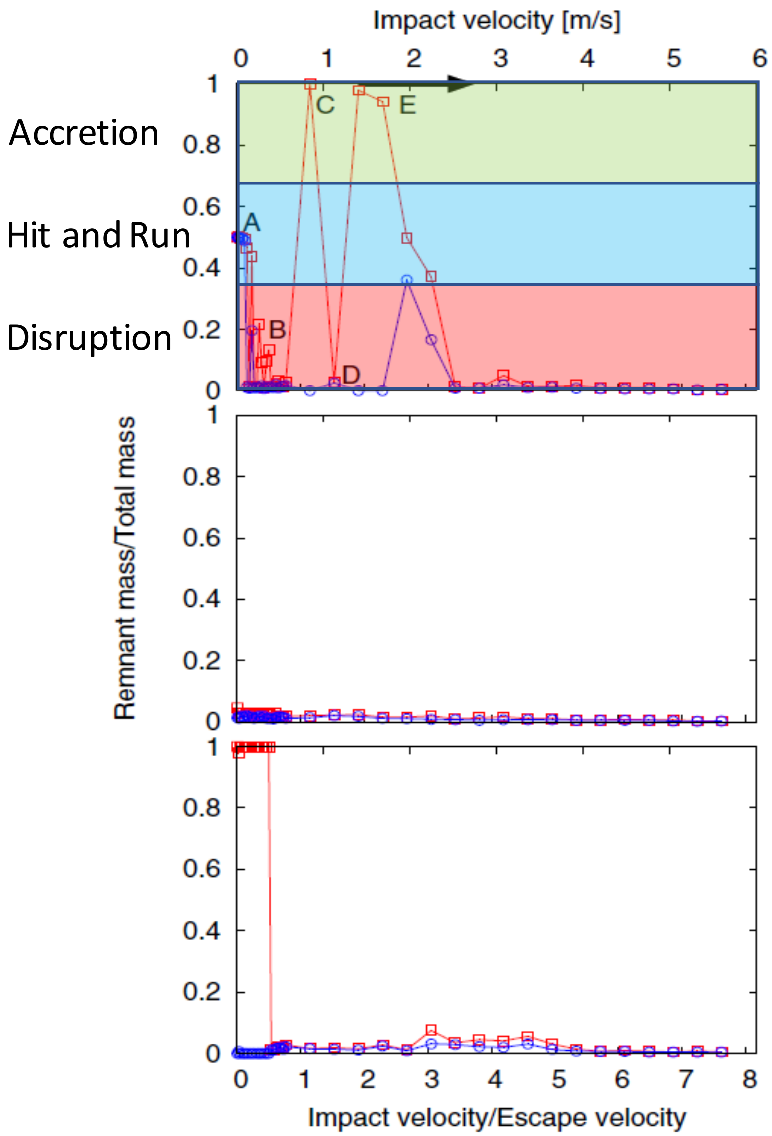
Using Numerical simulations results

- What happens when equal sized object collide randomly?
- Can Numerical simulations be used to base the statistics of random events?

Using the results of Hyodo and Ohtsuki:

- The outcomes of the stochastic events are based on the ratio of Impact/Escape velocity and the direction of collision.
- The direction of collision can be radial, azimuthal and vertical. Direction is chosen with equal probability.





Hyodo and Ohtsuki: 140K km case simulation

Random Event Outcomes:

- Accretion : Green region: This event doubles the current mass.
- Hit and run : Blue region: This event does not change the mass.
- disruption : Red region: This event halves the mass.

Note: This simulation considers presence of strong tidal waves. (Distance from Saturn :140k km)

Limiting mass calculation: computed based on cell size

$$M_L = 500m \times 10^4 m \times \Sigma_0$$

$$M_L = 5 \times 10^4 cm \times 10^6 cm \times 100 g / cm^2$$

$$M_L = 5 \times 10^{12} g \; or \; 5 \times 10^9 kg$$

$$M_0 = 1.0472 \times 10^6 kg$$

$$M_L = 4.7746 \times 10^3 \mathrm{M}_0 \sim 5 \times 10^3 M_0$$

Input parameters:

tauS = 1

tauB = 0.1 ; tauS/tauB = 10.

epsilon = 0.1 Coefficient of restitution

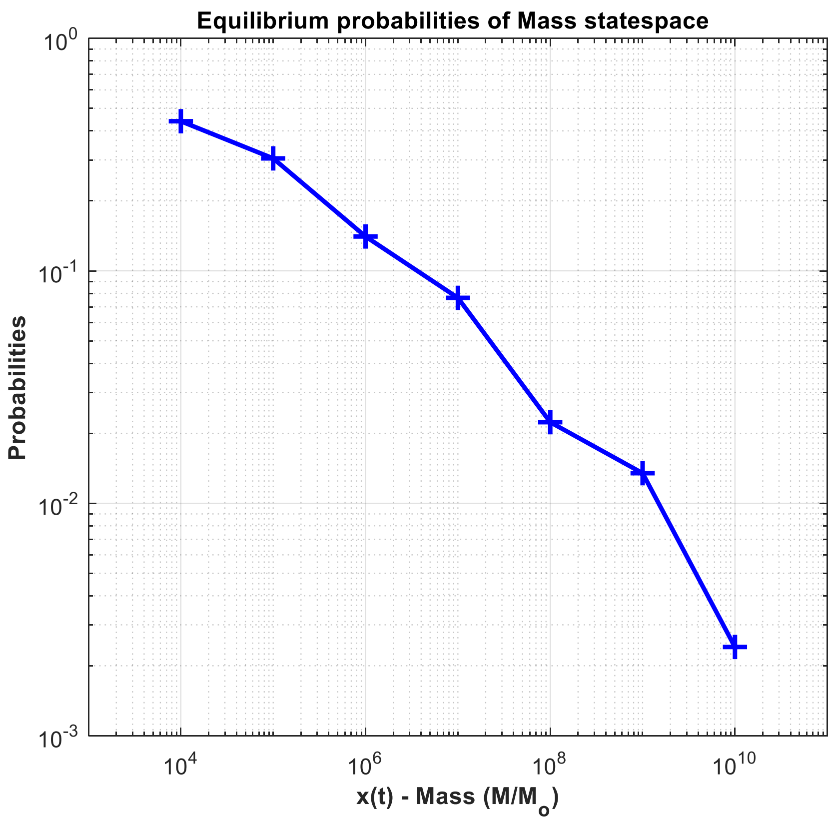
rho0 = 0.25g/cm^3. Uncompressed density of ring particle aggregates.

m0 = 1.05 x 10^9 g, mass of R0=10m sphere with rho0=0.25g/cm^3. Reference mass.

S = 300cm, small particle radius, from mass density rho0=0.25 g/cm^3, and optical depth tauS=0.1.

Vthresh(M0) = 1 cm/sec

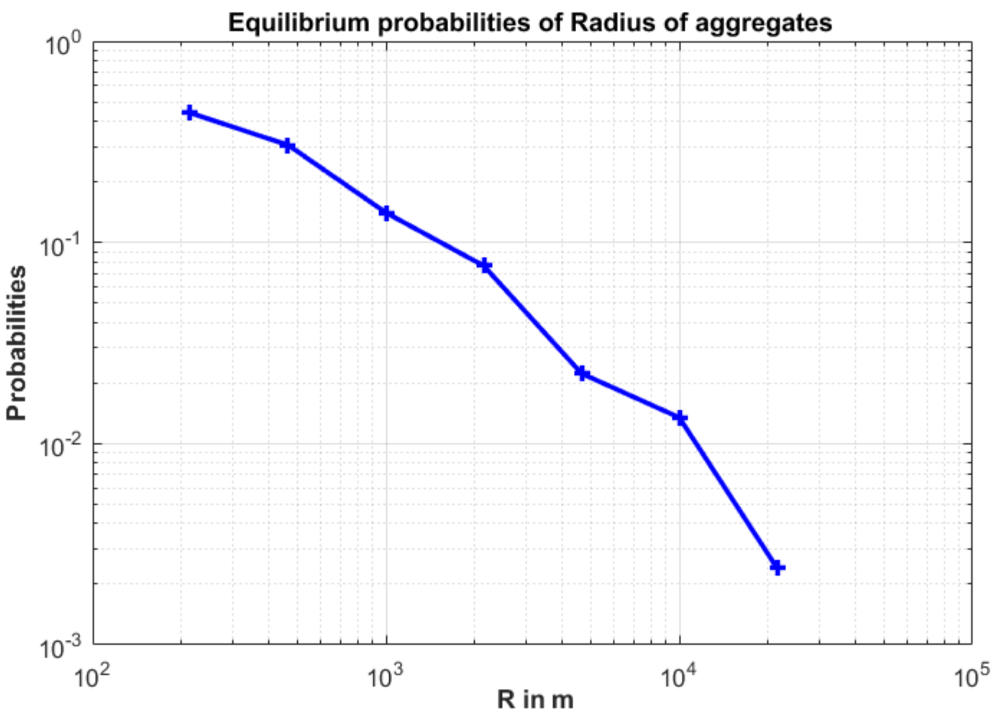
Equilibrium distribution of Mass of aggregates:



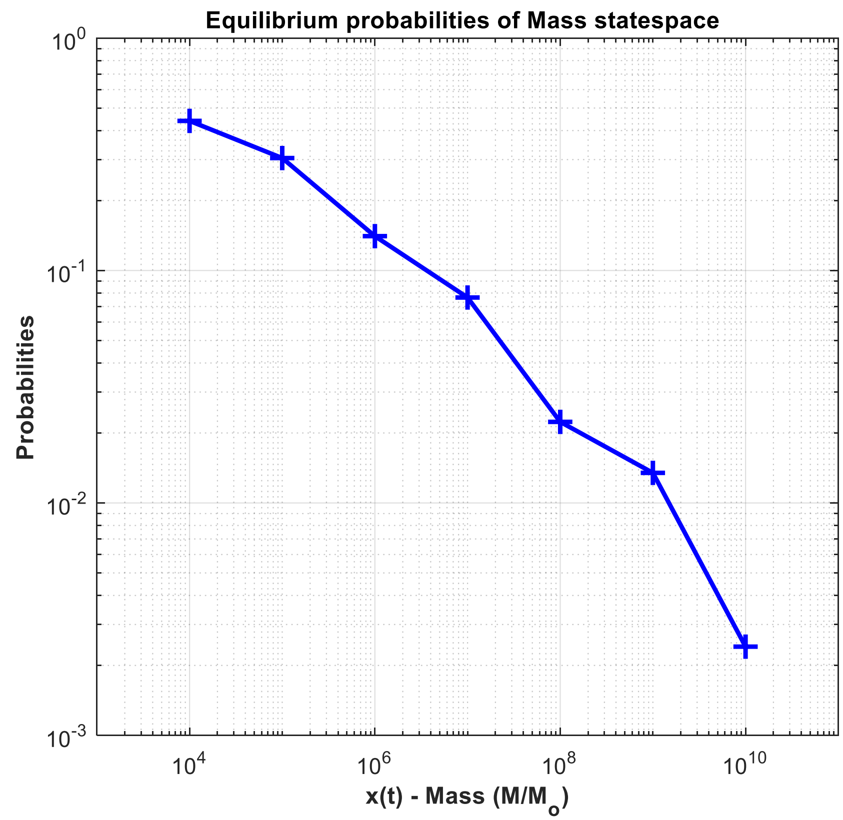
The Power Law for index radius distribution was found to be : -0.3386

Final Plots (Distance from Saturn 140K km, presence of tidal environment)

Power Law Index: -1.0158



Power Law Index: -0.3386



Conclusion:

- The Predator Prey model can include the outcomes of random collisions in the presence of tidal environment by using the results of numerical simulations.
- The power law index of mass distribution was found to be : -0.3386
- The power law index of radius of aggregates distribution was found to be : -1.0158
- The power law index of the mass distribution obtained from the simulation match well with results obtained from observations. (more explanation might be needed for this point)
- The Mass distribution can be computed for different settings of tidal environment.
- The Long term behavior of the rings can be statistically predicted using the equilibrium mass distributions using Predator Prey model, which could otherwise be very time consuming.
- Though there is a strong presence of tidal environment (140k km, there is still a possibility of finding aggregates with high masses, this could explain the presence of Straws in F ring ?(Not very sure about this point)